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Problems of Dynamics in Generally Covariant Quantum Field Theory

Hadi Salehi[1]

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Problems connected with the structural aspects of dynamics are addressed in the context of the algebraic approach to generally covariant quantum field theory. It is argued that the dynamical structure of observables in the generally covariant context becomes fundamentally state dependent. This makes it necessary to relate the entire dynamics to state-dependent automorphisms of the algebra of observables. The relevant states are highly correlated on large scales, so that we may not have exact accuracy for the identification of their observables in terms of a (quasi) local net of algebras. This feature is controlled by a scale fluctuation of the total observables around a point which is used to obtain a description of a one-parameter group of state-dependent automorphisms in terms of the modular group. In general, it is not clear whether the action of the latter group has a dynamical interpretation. We comment on a duality principle which could provide a straightforward means to obtain an "asymptotic" interpretation of the modular group on small scales.

1. INTRODUCTION

The formulation of a quantum field theory incorporating the basic structures of general relativity is widely considered to represent the key problem of quantum gravity. There has been a considerable amount of work approaching this problem, but at present it seems to be fair to say that there is no general agreement on choosing the basic principles on which the theory should operate.

Quantum field theory, in essence, originated from the attempt at a unification of special relativity and quantum physics. The present approach to this unification is based on the principle of locality, which asserts that the physical

[1] Institute for Studies in Theoretical Physics and Mathematics, Niavaran-Tehran, Iran, and Arnold Sommerfeld Institute for Mathematical Physics, TU Clausthal, D-38678 Clausthal-Zellerfeld, Germany; e-mail: salehi@netware2.ipm.ac.ir.

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systems are fundamentally local systems in the sense that their observables can be measured in finite space-time regions and observables associated to causal disjoint regions are always compatible. This attitude was the starting point of the algebraic approach to quantum field theory (Haag and Kastler, 1964).

On the structural level, however, one encounters in this attitude a theoretical idealization dismissing a possible incorporation of gravity at the very outset. This idealization concerns the assumption of *a priori* causal relations between observables associated to distant space-time regions. A striking example of the kind of difficulty one encounters is provided by looking at the basic principle of general relativity, namely the principle of general covariance. Since the action of the group of space-time diffeomorphisms does not leave the causal relations unchanged, we would obtain a contradiction if we were to implement the general covariance *a priori*.

In dealing with this difficulty the central thesis is that the principle of locality ought to be advanced in its most stringent form dispensing with the existence of *a priori* causal relations between distant observables. It is along this line that one can hope for a consistent unification of gravity and quantum field theory.

This point of view has been adopted by Fredenhagen and Haag (1987) in their approach to a generally covariant quantum field theory. Their work seems to clarify considerably the question of how general covariance can be incorporated into quantum field theory.

In the present work we address the problem of specifying the structure of dynamics in generally covariant quantum field theory. The significance of this problem is obvious, as the incompatibility of general covariance with the existence of *a priori* algebraic relations between observables mentioned above confronts us with the crucial question of how a dynamical structure can properly be imposed on the observables. It is needless to say that the resolution of this problem is quite essential for understanding what can be observed and predicted in quantum gravity. However, it should clearly be understood that the present paper undertakes only an extremely modest attempt in this direction; its spirit is extremely heuristic and much work will be required to treat these problems on a rigorous basis.

First, we present a description of the algebraic approach to generally covariant quantum field theory (Fredenhagen and Haag, 1987) on which our presentation is based (see also Salehi, 1992). We deal with a differentiable manifold M and associate to each open set $\mathcal{O} \in \mathcal{M}$ an involutive algebra $\mathcal{A}(\mathcal{O})$. The self-adjoint elements of $\mathcal{A}(\mathcal{O})$ are interpreted as observation procedures, the latter being pure descriptions of laboratory measurements in $\mathcal{O}$. There should not be any *a priori* relations between observation procedures associated with different regions. In other words, the net of algebras $\mathcal{A} = \bigcup \mathcal{A}(\mathcal{O})$ has to be flexible (free from *a priori* relations).

This interpretation allows us to implement the principle of general covariance by considering the group *Diff*(M) of all diffeomorphisms of the manifold as acting by automorphisms on $\mathcal{A}$, i.e., each diffeomorphism $\chi \in$ *Diff*(M) is represented by an automorphism α_χ such that

$$\alpha_\chi(\mathcal{A}(\mathcal{O})) = \mathcal{A}(\chi(\mathcal{O})) \tag{1}$$

There could be, of course, many observation procedures which are equivalent with respect to their action on a physical system, that is, with respect to the result of measuring an observable. Thus, the essential question is how to identify the observables of the theory as equivalence classes of observation procedures. To this aim, we first note that the precise mathematical description of a physical system is given in terms of a state, i.e., a positive linear functional on $\mathcal{A}$. Given a state ω, one gets via the GNS construction a representation π^ω of $\mathcal{A}$ by an operator algebra in a Hilbert space $\mathcal{H}^\omega$ with a cyclic vector $\Omega^\omega \in \mathcal{H}^\omega$. In the representation $(\tau^\omega, \mathcal{H}^\omega, \Omega^\omega)$ one can select a family of related states on $\mathcal{A}$, namely those represented by vectors and density matrices in $\mathcal{H}^\omega$. It corresponds to the set of normal states of the representation π^ω, the so-called folium of ω.

Once a physical state ω has been specified, one can consider in each subalgebra $\mathcal{A}(\mathcal{O})$ the equivalence relation

$$A \sim B \leftrightarrow \omega'(A - B) = 0, \qquad \forall\omega' \in \mathcal{F}^\omega \tag{2}$$

Here $\mathcal{F}^\omega$ denotes the folium of the state ω. The set of such equivalence relations generates a two-sided ideal $\mathcal{J}^\omega(\mathcal{O})$ in $\mathcal{A}(\mathcal{O})$. Now, the algebra of observables $\mathcal{A}^\omega_{\text{obs}}(\mathcal{O})$ may be constructed from the algebra of observation procedures $\mathcal{A}(\mathcal{O})$ by taking the quotient

$$\mathcal{A}^\omega_{\text{obs}}(\mathcal{O}) = \mathcal{A}(\mathcal{O})/\mathcal{J}^\omega(\mathcal{O}) \tag{3}$$

It is clear that in this approach the emphasis in the specification of physical laws, i.e., relations between observables of the theory, is placed on the specification of admissible physical states. In this way the dynamical structures of observables become fundamentally state dependent. This has a deep dynamical significance because it asserts that it is the intrinsic structure of physical states that will determine the allowed structure of observables in the generally covariant context. Our main objective in rest the of the paper is to examine the extent to which this idea could find a well-established implementation.

For mathematical convenience in what follows we pass[2] from the net of the algebra of observables $\mathcal{A}^{\omega}_{\text{obs}}(\mathbb{O})$ to the corresponding net of von Neumann algebras $\mathcal{R}^{\omega}_{\text{obs}}(\mathbb{O})$.

At this point a remark is needed. Since in the generally covariant context no diffeomorphism-invariant notion for localization exists, it is not clear in what sense the net $\mathcal{R}^{\omega}_{\text{obs}}(\mathbb{O})$ can be considered as local. However, because of the existence of physical laws in that net some structural aspects of localizations may be converted into appropriate algebraic properties of the net. Indeed, if we realize that the physical meaning of localization requires the nonnegligible existence of relative correlations of "the outside," then, in a minimal adoption of the locality principle, we must restrict the theory to a net $\mathcal{R}^{\omega}_{\text{obs}}(\mathbb{O})$ for which the commutant $\mathcal{R}^{\omega\prime}_{\text{obs}}(\mathbb{O})$ is nontrivial. A region $\mathbb{O} \subset M$ satisfying this condition will be called in the following a test region. Thus, in the minimal adoption of the locality principle, we may confine our attention to a net spanned by algebras corresponding to test regions.

2. DYNAMICAL GROUP

We want now to investigate the implementation of general covariance more closely. The basic postulate is that to each diffeomorphism $\chi \in \mathit{Diff}(M)$ there is associated an automorphism α_χ such that (1) holds.

Consider a diffeomorphism $\chi \in \mathit{Diff}(M)$. Given a state ω, we may ask whether the action of χ on the algebra of observables can be described by the same automorphism α_χ, namely

$$\alpha_\chi(\mathcal{R}^{\omega}_{\text{obs}}(\mathbb{O})) = \mathcal{R}^{\omega}_{\text{obs}}(\chi(\mathbb{O})) \tag{4}$$

In order for this to be possible, the ideal $\mathcal{J}^{\omega}(\mathbb{O})$ must transform covariantly, i.e.,

$$\alpha_\chi(\mathcal{J}^{\omega}(\mathbb{O})) = \mathcal{J}^{\omega}(\chi(\mathbb{O})) \tag{5}$$

We infer that the algebra of observables constructed with respect to the folium of the state ω no longer exhibits the basic symmetry of the theory, namely the symmetry under the whole group $\mathit{Diff}(M)$. Rather the symmetry is now reduced to the group of diffeomorphisms satisfying the constraint condition (5), in the following called the dynamical group of ω. Therefore, the actual choice of the state ω immediately leads to a spontaneous breaking of the symmetry group $\mathit{Diff}(M)$. Note that the dynamical group of the state ω arises as an intrinsic property of the folium of ω.[3]

[2] The transition from $\mathcal{A}^{\omega}_{\text{obs}}(\mathbb{O})$ to $\mathcal{R}^{\omega}_{\text{obs}}(\mathbb{O})$ is explained in Fredenhagen and Haag (1987).

[3] One cannot say with confidence that this feature is always free from physical inconsistencies in a general covariant context, for it is possible that the individual characteristics of the states in the same representation may become important in the construction of dynamics. It is entirely open to what extent this aspect of the dynamics, which is related to the problem of backreaction (Salehi, 1992), will affect the structure of generally covariant quantum field theories.

It is clear that an important step in the actual specification of a physically admissible state in a general covariant context is the understanding of the action of the corresponding dynamical group. This problem has two distinct aspects: first, the specification of the algebraic action of the dynamical group as a group of state-dependent automorphisms acting on observables; second, the geometric interpretation of the latter group. It is clear that, not knowing absolute forms of geometric symmetries in a general covariant context, it is necessary to introduce the above distinction between the algebraic and the geometric aspects of the problem. The present paper deals mainly with the first aspect. Our basic remark is that for physically admissible states the action of the dynamical group corresponds to global operations affecting the entire net of the algebra of observables. The need for this limitation is obvious, as in the generally covariant context the origin of the dynamical group ought to be attributed to the inertial manifestation (dynamically ascertainable properties) of observables, and if, according to the implications of general covariance, inertia is to be understood as a global dynamical effect of a closed physical system, we must then require that the local action of the dynamical group about a given point of observation be correlated with a universal action affecting the entire net of observables around that point.

If this is to be assumed, then for physically admissible states the action of the dynamical group cannot properly be approximated by the action of an inner automorphism of the algebra of observables. Indeed, the assumption that α_χ be generated by a "localized" element U in the test-region $\mathcal{O}_0$, namely

$$\alpha_\chi(A) = UAU^{-1} \tag{6}$$

implies that the action of α_χ on the commutant of $\mathcal{R}^{\omega}_{\text{obs}}(\mathcal{O}_0)$ can properly be approximated by the identity. This is in opposition to the assumption that the dynamical group has a global action.

It should clearly be understood that the global action of the dynamical group being assumed here requires two things. On the one hand, there shall be large-scale correlations in the structure of physically admissible states, a property typical of quantum field theory. Second (and this is supposed to be the distinguishing characteristic of general covariance), there shall be no possibility for the exact separation of the net of the algebra of total observables around a given point of observation from the effect of large-scale correlations which exist in the structure of physical states.[4]

Mathematically, these qualitative requirements can be converted into appropriate restrictions on the representations of the algebra of observables

[4] Note that in Minkowski space, due to the cluster property of Wightman functions (Haag, 1992), it is possible to have exact separation of the entire net of observables from the "asymptotic tail" of long-distance correlations existing in the vacuum.

in a general covariant context. First, consider the cyclic vector $\Omega^{\omega} \in \mathcal{H}^{\omega}$ in the GNS representation of a physical state ω, then pick a test region $\mathcal{O} \subset \mathcal{M}$ and ask whether Ω^{ω} can be annihilated by elements of $\mathcal{R}^{\omega}_{\text{obs}}(\mathcal{O})$ [respectively, by elements of $\mathcal{R}^{\omega\prime}_{\text{ob}}(\mathcal{O})$]. It is obvious that, if the existence of large-scale correlations in the structure of physical states is considered to be the basic attribute of general covariance, then a proper separation of the physical process of annihilation of Ω^{ω} by elements in $\mathcal{R}^{\omega}_{\text{obs}}(\mathcal{O})$ [respectively by elements of $\mathcal{R}^{\omega\prime}_{\text{obs}}(\mathcal{O})$] from the effect of large-scale correlations which exists in ω must be impossible. Mathematically, this may be expressed by admitting any test region $\mathcal{O} \subset \mathcal{M}$ to have the vector Ω^{ω} as a separating vector for both $\mathcal{R}^{\omega}_{\text{obs}}(\mathcal{O})$ and $\mathcal{R}^{\omega\prime}_{\text{obs}}(\mathcal{O})$, that is,

$$A\Omega = 0, \qquad A \in \mathcal{R}^{\omega}_{\text{obs}}(\mathcal{O}) \qquad [\text{resp. } A \in \mathcal{R}^{\omega\prime}_{\text{obs}}(\mathcal{O})] \tag{7}$$

implies $A = 0$. Now standard arguments (Bratteli and Robinson, 1981) may be used to prove that Ω^{ω} must be both a cyclic and a separating vector for $\mathcal{R}^{\omega}_{\text{obs}}(\mathcal{O})$.

It is now clear that, in a minimal adoption of the locality principle, if we require that the net of algebras is to be spanned by algebras corresponding to test regions, then for the restriction of ω on each test region $\mathcal{O} \subset M$ the vector Ω^{ω} must be a cyclic and separating vector for $\mathcal{R}^{\omega}_{\text{obs}}(\mathcal{O})$. This is a first restriction imposed on the kind of representation by the requirement of general covariance.

It should, however, be realized that this restriction does not yet explicitly take into account the distinguishing characteristic of general covariance, the latter asserting that, given a point $x \in M$, there shall be no possibility for the exact separation of the net of the algebra of observables around that point from the effect of large-scale correlations. To deal with it, let $D^x(M)$ denote the set of all test regions in M containing x as an interior point. Now, suppose we are given, quite in the sense of an additive net structure, an increasing sequence $\mathcal{O}^x_n \subset \mathcal{O}^x_{n+1}$ of test regions in $D^x(M)$ to span the net of the algebra of observables around x. We may then ask whether Ω^{ω} can be annihilated by an element in the algebra corresponding to asymptotic tail ($n \to \infty$) of the sequence. It is clear that, in the absence of an exact separation of the net of the algebra of observables around x from the effect of large-scale correlations, we should reject the admission of this kind of annihilation. This means that the asymptotic tail of the sequence should have properties arbitrarily close to a test region, so that for the restriction of ω on the latter region, Ω^{ω} still remains a cyclic and separating vector. Therefore we may, mathematically, require that any appropriate increasing sequence of test regions needed to span the net of total observables around x should correspond to the elements of a partially (with respect to the inclusion $\subseteq$) ordered set, with a maximal element corresponding to the asymptotic tail of that sequence. This is a statement about the distinguishing characteristic of the net structure of observables in a general covariant context.

3. CHARACTERISTIC DOMAIN OF STATES AROUND A POINT

The structural properties of the net of the algebra of observables which are expected to hold in a general covariant context have been discussed in the previous section. Given a point of observation, since no exact separation of the net of the algebra of observables around that point from the effect of large-scale correlations is possible, there is the important question of how the net can dynamically be closed around that point. In this section we take the key technical step to deal with this question. The major procedure we shall follow is, in the first place, to look more closely at the expected characteristics of states with large-scale correlations, and in the second place, to use these characteristics to explain how the net of the algebra of observables around a given point of observation can dynamically be closed.

The expected characteristics of states with large-scale correlations in a general covariant context is that they cannot maintain enough accuracy for the exact identification of their observables around a point in terms of a (quasi) local net of algebras. Therefore, they are expected to give rise to the occurrence of nonnegligible relative fluctuations in that net which makes the observables determined only with a finite accuracy. As 'relative' fluctuations in a system are expected to increase as the size of the systems becomes progressively larger, we may arrive at the conclusion that, given a state with large-scale correlations in a general covariant context, it is possible and warranted to deal with a numerical truncation in the global extension of the corresponding (quasi) local net of observables around an arbitrary point of observation. This aspect is clearly reflected in the particular way we use to control the asymptotic tail of any appropriate directed sequence of test regions needed to span the net of observables around a point. The essential realization now is that, physically, the effect of this type of truncation may be equivalent to the appearance of a minimal observable scale which may be looked upon as representing the nonvanishing lower bound for the effective ratio of an appropriately chosen local (small) length to a global (large) length. This scale must be related to the presence of the absolute form of the restrictions on the maximal achievable limit of accuracy for the exact determination of total observables around a given point and in this sense it must be identified as a common property of the states in the same representation.

The mathematical counterpart of this observation is the following. Let $x \in M$ be fixed. We just assume that a characteristic scale $\lambda(x)$ can be attached to x once a physical state ω and its GNS representation $(\mathcal{H}^{\omega}, \pi^{\omega}, \Omega^{\omega})$ have been specified.[5] This scale can be related to the structural properties of the net of the algebra of observables around x: Consider the set $D^x(M)$ of all test regions in M containing x as an interior point. Let $\mathcal{O}_n^x \subset \mathcal{O}_{n+1}^x$ be an increasing sequence of test regions in $D^x(M)$ needed to span the net of the algebra of observables around x. Denoting its maximal element by $\mathcal{O}_\infty^x \in D^x(M)$, we shall assume that $\lambda(x)$ corresponds to the maximal accuracy by which all states in the folium of ω can be regarded as indistinguishable in their canonical restriction to the commutant of $\mathcal{R}_{\text{obs}}^\omega(\mathcal{O}_\infty^x)$, namely[6]

$$\|(\omega' - \omega'')|_{\mathcal{R}_{\text{obs}}^{\omega'}(\mathcal{O}_\infty^x)}\| \simeq \lambda(x), \qquad \forall \omega', \omega'' \in \mathcal{F}^\omega \tag{8}$$

In terms of observables, this condition implies that, with the maximal accuracy determined by $\lambda(x)$, the total algebra of observables around the point x can be identified with the algebra $\mathcal{R}_{\text{obs}}^\omega(\mathcal{O}_\infty^x)$. In this sense, the presence of $\lambda(x)$ implies an absolute form of restrictions on the maximal achievable limit of accuracy for the exact determination of total observables around a given point.

In the following we shall call $\mathcal{O}_\infty^x$ the characteristic domain of ω around the point x. Note that a physical state ω in its natural restriction to $\mathcal{O}_\infty^x$ admits an algebra of observables $\mathcal{R}_{\text{obs}}^\omega(\mathcal{O}_\infty^x)$ with a nontrivial commutant $\mathcal{R}_{\text{obs}}^{\omega'}(\mathcal{O}_\infty^x)$.

We now deal with the question of the influence of $\lambda(x)$ on the nature of the action of the dynamical group on observables. There is a natural way to account for this influence. Indeed, as the maximal accuracy in the determination of the total observables around a given point is limited by λ, it is natural to require that this kind of limitation be respected by the action of the dynamical group on observables. In specific terms this may lead us to require the conditions

$$\alpha_\chi(\mathcal{R}_{\text{obs}}^\omega(\mathcal{O}_\infty^x)) \subseteq \mathcal{R}_{\text{obs}}^\omega(\mathcal{O}_\infty^x), \qquad \alpha_\chi(\mathcal{R}_{\text{obs}}^{\omega'}(\mathcal{O}_\infty^x)) \subseteq \mathcal{R}_{\text{obs}}^{\omega'}(\mathcal{O}_\infty^x) \tag{9}$$

which express a sort of dynamical stability. The conditions mean in particular that the algebra $\mathcal{R}_{\text{obs}}^\omega(\mathcal{O}_\infty^x)$ can be regarded as dynamically closed around x. It is clear that this is a statement about the particular way in which the dynamical group acts geometrically on the observables. It roughly means that this action does not move the points to "infinity."

4. MODULAR GROUP

Generally, the dynamical group of an admissible physical state ω may have various subgroups. These subgroups will describe the symmetry transfor-

[5] It is not the objective of this paper to make any assumptions about the (average) value of $\lambda(x)$. We uncritically accept its existence in any realistic situation in a general covariant context.
[6] In the following the norm of a state ω corresponds to the smallest possible number K for which $|\omega(A)| < K\|A\|$ holds for an arbitrary element A of the algebra in question.

mations respecting the algebraic relations imposed by the state ω on the observables of the theory. Therefore, further restrictions on the class of allowed representations of the algebra of observables can be imposed by specifying the allowed subgroups. The problem is by no means simple, because in the present context the identification of various subgroups requires detailed knowledge of the state-dependent dynamics involved. In the rest of the paper we shall deal with some isolated aspects of the problem which may significantly be connected with the modular group.

We first start with the definition of the modular group as a one-parameter group of state-dependent automorphisms on $\mathcal{R}^{\omega}_{\rm obs}(\mathcal{O}^x_\infty)$: Let $x \in M$ be fixed. Given an admissible physical state ω in its natural restriction to its characteristic domain $\mathcal{O}^x_\infty$ around x, we may use the GNS representation $(\mathcal{H}^\omega, \pi^\omega, \Omega^\omega)$ of ω to get a characterization of the observables on the characteristic domain of ω in terms of a (quasi) local net of Neumann algebras

$$\mathcal{R}^{\omega}_{\rm obs}(\mathcal{O}^x_\infty) = \cup\, \mathcal{R}^{\omega}_{\rm obs}(\mathcal{O}), \qquad \mathcal{O} \subset \mathcal{O}^x_\infty \tag{10}$$

Now, in the present context the canonical procedure of Tomita and Takesaki (Bratteli and Robinson, 1981; Haag, 1982) may be adopted to associate with $\mathcal{R}^{\omega}_{\rm obs}(\mathcal{O}^x_\infty)$ a one-parameter group of (outer) automorphisms. We collect here the essential steps. Since Ω^ω is a cyclic and separating vector for $\mathcal{R}^{\omega}_{\rm obs}(\mathcal{O}^x_\infty)$, we can consider the operators S and F defined by

$$SA\Omega^\omega = A^*\Omega^\omega, \qquad A \in \mathcal{R}^{\omega}_{\rm obs}(\mathcal{O}^x_\infty) \tag{11}$$

$$FA'\Omega^\omega = A'^*\Omega^\omega, \qquad A' \in \mathcal{R}^{\omega\prime}_{\rm obs}(\mathcal{O}^x_\infty) \tag{12}$$

One can show that S and F are closed operators. Let

$$S = J\Delta^{1/2} \tag{13}$$

be the polar decomposition of S. Here J is antiunitary and Δ, the so-called modular operator, is self-adjoint and positive (one has $\Delta = FS$). The Tomita–Takesaki theorem provides us with a one-parameter group of automorphisms α_t on $\mathcal{R}^{\omega}_{\rm obs}(\mathcal{O}^x_\infty)$, the modular group of ω, defined by

$$\alpha_t(A) = \Delta^{-it}A\Delta^{it}, \qquad A \in \mathcal{R}^{\omega}_{\rm obs}(\mathcal{O}^x_\infty) \tag{14}$$

We may conclude that, restricting a state ω to its characteristic domain, we meet the canonical action of the modular group of ω as a group of state-dependent automorphisms on $\mathcal{R}^{\omega}_{\rm obs}(\mathcal{O}^x_\infty)$. At the first instance, it appears as natural to require that the latter action shall coincide with the action of a one-parameter subgroup of the dynamical group of ω. Indeed, doing so, we would be able to connect the effect of large-scale correlations with the thermal behavior of states,[7] a feature which will be of particular importance for the interpretation of the theory in physical terms.

However, the above requirement is a very strong limitation, because it requires that the modular automorphisms on $\mathcal{R}^{\omega}_{\text{obs}}(\mathcal{O}^x_\infty)$ have a somewhat global interpretation on the whole characteristic domain of ω, and the question arises: Why we should believe in it? It is important to note that the confidence that one feels in that requirement arises because it leads to thermal properties of states. However, in a general covariant context, thermal properties should be local properties. Therefore, it seems more natural to avoid global considerations and to deal with the less restrictive requirement that the modular group shall have an asymptotic interpretation on a "strictly localized" subdomain of $\mathcal{O}^x_\infty$. To proceed in this direction, we introduce a duality principle.

5. DUALITY AND ULTRASHORT DISTANCES

So far we have relied on the idea that it is impossible to separate the net of the algebra of observables around a given point x from the effect of large-scale correlations which exists in the structure of physical states in a general covariant context. We would like to argue that this kind of description can exhibit aspects of ignorance of the "ultrashort-distance regime." The essential input here is to exclude the possibility of dealing with pointlike observables, that is, observables whose measurements require arbitrarily high energies. In the present context there is one particular way to do this. Since the effect of large-scale correlations may in principle lead to thermal behavior of states, it appears that, given an observable, its strict localization may result in a transfer of that observable into thermal entropy. By using this heuristic idea, we propose restrictive conditions concerning the minimization of the net of the algebra of observables. We first assume that the minimization of the latter net around a given point x will amount to an algebra corresponding to the asymptotic tail of a contracting sequence

$$\mathcal{O}^x_{n+1} \subset \mathcal{O}^x_n, \qquad x \in \mathcal{O}^x_n, \quad \mathcal{O}^x_n \subset \mathcal{O}^x_\infty \tag{15}$$

of test regions. We also accept that it is necessary for an adequate minimization that the latter sequence corresponds to elements of a partially ordered set with a minimal element, say $\mathcal{O}^x_0$. Now the essential point is that, in accordance with what has been heuristically suggested, we must have maximal lack of information about the elements of the algebra $\mathcal{R}^{\omega}_{\text{obs}}(\mathcal{O}^x_0)$. Now, as the total algebra of observables around x is identified with $\mathcal{R}^{\omega}_{\text{obs}}(\mathcal{O}^x_\infty)$, we may require the duality relation

[7] Note that any faithful state is a KMS state with respect to the modular group it generates (Bratteli and Robinson, 1981; Haag, 1992).

$$\mathcal{R}_{obs}^{\omega}(\mathcal{O}_0^x) = \mathcal{R}_{obs}^{\omega\prime}(\mathcal{O}_\infty^x) \tag{16}$$

which connects in a very restrictive manner the physics of ultrashort distances with the effect of large-scale correlations.

It should be remarked that this duality has a counterpart in standard results of quantum field theory in Minkowski space. There, due to the exact Lorentz invariance, the algebra of the spacelike complement of a single point generates the total algebra. In the present context, the relation (16) stands in close analogy with that statement and is taken to be one of the basic features of general covariance.

Now, the duality (16) is conjunction with (9) implies

$$\alpha_\chi(\mathcal{R}_{obs}^{\omega}(\mathcal{O}_0^x)) \subseteq \mathcal{R}_{obs}^{\omega}(\mathcal{O}_0^x) \tag{17}$$

which expresses the dynamical stability of $\mathcal{R}_{obs}^{\omega'}(\mathcal{O}_0^x)$. We can also relate the duality (16) to a statement regarding the local equivalence of states in the same representation. Indeed connecting (8) with (16), we get

$$\|(\omega' - \omega'')|_{\mathcal{R}_{obs}^{\omega}(\mathcal{O}_0^x)}\| \simeq \lambda(x) \qquad \forall \omega', \omega'' \in \mathcal{F}^{\omega} \tag{18}$$

This means that within the accuracy determined by $\lambda(x)$ all states in the folium of ω become indistinguishable on $\mathcal{O}_0^x$. We have, therefore, in (18) a sort of local equivalence of states in the same representation. It should be noted that in a fixed gravitational background the local equivalence of states in the same representation was attributed to the principle of local definiteness (Haag *et al.*, 1984). In the present context, the duality (16) attributes the origin of this equivalence to a principal ignorance of the ultrashort-distance regime.

The fact that all states in the folium of ω become physically indistinguishable on $\mathcal{O}_0^x$ suggests that they all correspond to the same local equilibrium state. This leads us to require that in the restriction of ω on the domain $\mathcal{O}_0^x$ the coincidence of the modular group[8] with a one-parameter subgroup of the dynamical group of ω shall be an exact feature in any generally covariant quantum field theory. This inevitably faces us with the important question about the geometric interpretation of the action of the modular group. It is not the objective of the present paper to clarify this question; still, some remarks may be useful. On general grounds we expect that there are restrictions imposed on the geometric action of the modular group on ultrashort distances by certain characteristics known from Minkowski space theories. For instance, in those theories, due to the observation of Hilsop and Longo (1982) (see also Fredenhagen, 1985), in the presence of a free massless scalar field, the modular group of a double cone is related to a one-parameter

[8] Note that the action of the modular group as a group of state-dependent automorphisms can be defined on $\mathcal{O}_0^x$ by replacing in (11)–(14) the set $\mathcal{O}_\infty^x$ by $\mathcal{O}_0^x$.

subgroup of the conformal group which has a timelike generator. Since on ultrashort distances a general theory is expected to approach its "massless sector," we expect that a similar analysis can be made in the generally covariant context, provided the massless sector is dominated by a scalar field theory (dilatonic sector).

6. CONCLUDING REMARKS

The present paper has described possible limitations arising from the effect of general covariance on the dynamical structure of observables in quantum field theory. These limitations give rise to state-dependent laws and are clearly reflected in the nature of restrictions imposed on the class of allowed representations. Of particular importance is the realization that the implementation of general covariance may lead to a nonnegligible scale fluctuation in the exact determination of the total observables around an arbitrary point of observation. This fluctuation has its origin in the structure of physical states, which are globally too correlated in order to maintain enough accuracy for the exact description of total observables by means of a (quasi) local net of algebras. This would restrict the nature of the dynamics quite essentially. Specifically, on ultrashort distances an intimate connection of the dynamics with the modular group may be a general phenomenon, a feature which is closely related to the thermal-time hypothesis recently proposed in Connes and Rovelli (1994). Needless to say, such a connection may also provide a general perspective for understanding the thermal aspects of quantized fields in a gravitational context, i.e., Unruh's (1976) Hawking's (1975) effect.

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